$c_1 = -10 \qquad c_2 = -20 \qquad c_3 = 0 \qquad c_4 = 0 \qquad c_5 = 0$

$$A = \begin{pmatrix} -1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 5 & 3 & 0 & 0 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 15 \\ 12 \\ 45 \end{pmatrix}$$

Select $x_3 \quad x_4 \quad x_5$ as basic variables. $\qquad x_B = \begin{pmatrix} x_3 \\ x_4 \\ x_5 \end{pmatrix}$

$x_B = b \qquad x_B = \begin{pmatrix} 15.00 \\ 12.00 \\ 45.00 \end{pmatrix}$

$x_3 = x_{B_1} \qquad x_4 = x_{B_2} \qquad x_5 = x_{B_3}$

$x_1 = 0 \qquad x_2 = 0$

Form matrix B $\qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad c_B = \begin{pmatrix} c_3 & c_4 & c_5 \end{pmatrix}$

$$a_1 = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix} \qquad a_2 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \qquad x_B = \begin{pmatrix} x_3 \\ x_4 \\ x_5 \end{pmatrix} \qquad x_N = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$w = c_B \times B^{-1} \qquad w = (0.00 \quad 0.00 \quad 0.00)$

$z_1 = w \times a_1 \qquad z_1 - c_1 = 10.00$

$z_2 = w \times a_2 \qquad z_2 - c_2 = 20.00$

Since, $z_2 - c_2 = 20.00$ is most positive, x_2 will enter solution

Calculate y_1

$$y_2 = B^{-1} \times a_2 \qquad y_2 = \begin{pmatrix} 2.00 \\ 1.00 \\ 3.00 \end{pmatrix}$$

Also, find new values of b; $b_n = B^{-1} \times b$ $\quad b_n = \begin{pmatrix} 15.00 \\ 12.00 \\ 45.00 \end{pmatrix}$

$$\text{ratio} = \begin{pmatrix} 7.50 \\ 12.00 \\ 15.00 \end{pmatrix}$$

Minimum value =7.5

x_3 will leave basic solution. Update the solution;

New matrix is $B = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$ $\qquad Y = B^{-1}$

$$Y = \begin{pmatrix} 0.50 & 0.00 & 0.00 \\ -0.50 & 1.00 & 0.00 \\ -1.50 & 0.00 & 1.00 \end{pmatrix} \qquad b_n = B^{-1} \times b \qquad b_n = \begin{pmatrix} 7.50 \\ 4.50 \\ 22.50 \end{pmatrix} \qquad x_B = \begin{pmatrix} x_2 \\ x_4 \\ x_5 \end{pmatrix}$$

$$x_B = b_n \qquad x_B = \begin{pmatrix} 7.50 \\ 4.50 \\ 22.50 \end{pmatrix}$$

$$x_2 = x_{B_1} \qquad x_2 = 7.50$$

$$x_4 = x_{B_2} \qquad x_4 = 4.50$$

$$x_5 = x_{B_3} \qquad x_5 = 22.50$$

$$c_B = \begin{pmatrix} c_2 & c_4 & c_5 \end{pmatrix} \qquad z_0 = c_B \times b_n \qquad z_0 = -150.00$$

OPTIMALITY TEST:

$$w = c_B \times B^{-1} \qquad w = \begin{pmatrix} -10.00 & 0.00 & 0.00 \end{pmatrix}$$

$$z_1 = w \times a_1 \qquad z_1 - c_1 = 20.00$$

Since, $z_1 - c_1 = 20.00$ is positive, x_1 will enter solution

Calculate y_1

$$y_1 = B^{-1} \times a_1 \qquad y_1 = \begin{pmatrix} -0.50 \\ 1.50 \\ 6.50 \end{pmatrix}$$

Also, find new values of b; $b_n = B^{-1} \times b$ $\qquad b_n = \begin{pmatrix} 7.50 \\ 4.50 \\ 22.50 \end{pmatrix}$

$$\text{ratio} = \begin{pmatrix} -15.00 \\ 3.00 \\ 3.46 \end{pmatrix}$$

Minimum positive ratio is 3.00, for variable

x_4 will leave solution

New matrix is

$$B = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 1 & 0 \\ 3 & 5 & 1 \end{pmatrix} \qquad Y = B^{-1}$$

$$Y = \begin{pmatrix} 0.33 & 0.33 & 0.00 \\ -0.33 & 0.67 & 0.00 \\ 0.67 & -4.33 & 1.00 \end{pmatrix} \qquad b_n = B^{-1} \times b$$

$$b_n = \begin{pmatrix} 9.00 \\ 3.00 \\ 3.00 \end{pmatrix} \qquad x_B = \begin{pmatrix} x_2 \\ x_1 \\ x_5 \end{pmatrix}$$

$$x_B = b_n \qquad x_B = \begin{pmatrix} 9.00 \\ 3.00 \\ 3.00 \end{pmatrix}$$

$$x_2 = x_{B_1} \qquad x_1 = x_{B_2} \qquad x_5 = x_{B_3}$$

$$x_3 = 0 \qquad x_4 = 0$$

Hence, $x_1 = 3.00$ $x_2 = 9.00$ $x_3 = 0.00$

$x_4 = 0.00$ $x_5 = 3.00$

$$c_B = \begin{pmatrix} c_2 & c_1 & 0 \end{pmatrix} \qquad z_0 = c_B \times b_n$$

OPTIMAL SOLUTION $z_0 = -210.00$ $x_1 = 3.00$ $x_2 = 9.00$